

Characterization of large price variations in financial markets

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February 1, 2008

Abstract

Statistics of drawdowns (loss from the last local maximum to the next local minimum) plays an important role in risk assessment of investment strategies. As they incorporate higher ($>$ two) order correlations, they offer a better measure of real market risks than the variance or other cumulants of daily (or some other fixed time scale) of returns. Previous results have shown that the vast majority of drawdowns occurring on the major financial markets have a distribution which is well-represented by a stretched exponential, while the largest drawdowns are occurring with a significantly larger rate than predicted by the bulk of the distribution and should thus be characterized as *outliers* [1, 2]. In the present analysis, the definition of drawdowns is generalised to coarse-grained drawdowns or so-called ϵ -drawdowns and a link between such ϵ -*outliers* and preceding log-periodic power law bubbles previously identified [3] is established.

1 Introduction

The characterization of stock market moves and especially large drops, *i.e.*, large negative moves in the price, are of profound importance to risk management. A drawdown is defined as a persistent decrease in the price over consecutive days. A drawdown is thus the cumulative loss from the last maximum to the next minimum of the price, specifically the daily close in the analysis presented here. Since the definition of “maximum” and “minimum” is not unique except in a strict mathematical sense a drawdown may be defined in slightly varying ways. The definition used in the present paper is as follows. A drawdown is defined as the relative decrease in the price from a local maximum to the next local minimum *ignoring* price increases in between the two of maximum (relative or absolute) size ϵ . We will refer to this definition of drawdowns as “ ϵ -drawdowns”, where we will refer to ϵ as the *threshold*.

Drawdowns embody a rather subtle dependence since they are constructed from runs of the same sign variations. Their distribution thus captures the way successive drops can influence each other and construct in this way a (quasi-)persistent process. This persistence is not measured by the distribution of returns because, by its very definition, it forgets about the relative positions of the returns as they unravel themselves as a function of time by only counting their frequency¹. This is not detected either by the two-point correlation function, which measures an *average* linear dependence over the whole time series and as a consequence wash out features only occurring at special times.

Related to the characterisation of drawdowns in the financial markets is the concept of “outliers” [1, 2]. An outlier to some specific distribution may defined as a point which position deviates sufficiently from those of the bulk of the distribution, it “lies out”, as to arise suspicion that different processes are responsible for the generation of one hand the overall distribution and on the other hand the outlier. Actually, testing for “outliers” or more generally for a change of population in a distribution is a quite subtle problem. This subtle point is that the evidence for outliers and extreme events does not require and is not even synonymous in general with the existence of a break in the distribution of the drawdowns. An example of this comes from the distribution for the square of the velocity variations in shell models of turbulence. Naively, one would expect that the same physics apply in each shell layer (each scale) and, as a consequence, the distributions in each shell should be the same, up to a change of unit reflecting the different scale embodied by each layer. The remarkable conclusion of L’vov *et al.* [5] is that the distributions of velocity increment seem to be composed of two regions, a region of so-called “normal scaling” and a domain of extreme events, the “outliers”.

Other groups have recently presented supporting evidence that crash and rally days significantly differ in their statistical properties from the typical market days. Lillo and Mantegna investigated the return distributions of an ensemble of stocks simultaneously traded in the New York Stock Exchange (NYSE) during market days of extreme crash or rally in the period from January 1987 to December 1998 [6]. Out of two hundred distributions of returns, one for each of two hundred trading days where the ensemble of returns is constructed over the whole set of stocks traded on the NYSE, anomalous large widths and fat tails are observed specifically on the day of the crash of Oct. 19 1987, as well as during a few other turbulent days. Specifically, they show that the overall shape of the distributions is modified in crash and rally days. Closer to our claim that markets develop precursory signatures of bubbles of long time scales, Mansilla [7] has also shown, using a measure of relative complexity, that time sequences corresponding to “critical” periods before large market corrections or crashes have more novel informations with respect to the whole price time series than those sequences corresponding to periods where nothing happened. The conclusion is that the intervals where no financial turbulence is observed, that is, where the markets works fine, the informational contents of the price time series is small. In contrast, there seems to be significant information in the price time series associated with bubbles.

In a series of papers, the authors [3, 4, 8, 9, 10, 11, 12] have shown that on the FX and major stock markets, crashes are often preceded by precursory characteristics quantified by a log-periodic power law, specifically

$$p(t) = A + B(t_c - t)^z + C(t_c - t)^z \cos(\omega \ln(t_c - t) - \phi) \quad (1)$$

¹Realising this allows one to construct synthetic price data with the same return distribution by a reshuffling of the returns[4]

The results on the US stock markets have been confirmed by several independent groups [13, 14, 15]

Eq. (1) has its origin in a Landau-expansion type of argument and the underlying *Scaling Ansatz* is simply

$$\frac{dF(x)}{d \ln x} = \alpha F(x) + z F^2(x) \dots \quad (2)$$

which to first order leads to eq. (1) with an arbitrary choice of periodic function. The concept that only relative changes are important has a solid foundation in finance, but a detailed and rigorous derivation or justification for the Ansatz (2) has not been achieved. Instead the predictions that comes from applying this Ansatz, specifically those related to t_c , z and ω , to the data has been compared using different markets and time periods. Specifically, we have found that

$$\omega \approx 6.36 \pm 1.56 \quad z \approx 0.33 \pm 0.18$$

for over thirty crashes on the major financial markets, see figures 1 and 2. This and the analysis to be presented in the following leads to a consistent and coherent picture when combined with the outlier concept. A comment on figure 2 is necessary here. A fit with eq. (1) will often generate more than one solution. In general, the best fit in terms of the r.m.s. is also the most sensible solution in terms of estimating the *physical* variables z and ω as well as the most probable time t_c of the end of the bubble, see [3] for a more detailed discussion. However, for a few cases the two best fits are included in the statistics which explains the presence of the “second harmonics” around $\omega \approx 11.5$.

2 Statistics and identification of bubbles and drawdowns

Lately, an increasing amount of evidence that the largest negative market moves belongs to a different population than the smaller has accumulated [1, 2, 3]. Specifically, it was found that the cumulative distributions of drawdowns on the worlds major financial markets, *e.g.*, the U.S. stock markets, the Hong-Kong stock market, the currency exchange market (FX) and the Gold market are well parameterised by a stretched exponential

$$N(x) = A e^{-bx^z} \quad (3)$$

except for the 1% (or less) largest drawdowns. In general, it was found that the exponent $z \approx 0.8 - 0.9$ [2], see figures 3 and 4 for two examples. It is worth noting that only the distributions for the DJIA, the US\$/DM exchange rate and the Gold price exhibits clear outliers for the complement drawup distribution, whereas (all?) other markets shows a strong asymmetry between the tails of the drawdown and drawup distributions the latter having no outliers. The range of the exponent for the drawup distributions is also generally higher with $z \approx 0.9 - 1.05$ except for the FX and Gold markets [2].

In the previous analysis and identification outliers on the major financial markets [2] drawdowns (drawups) were simply defined as a continuous decrease (increase) in the closing value of the price. Hence, a drawdown (drawup) was terminated by any increase (decrease) in the price no matter how small. A rather natural question concerns the effect of thresholding on the distribution of drawdowns (drawups). There are two straightforward ways to define a thresholded drawdown (drawup): We may ignore increases (decreases) of a certain fixed magnitude (absolute or relative to the price) or we may ignore increases (decreases) over a fixed time horizon in both cases letting the drawdown (drawup) continue. We will refer this in general as coarse-grained drawdowns (drawups) due to the smoothing obtained by ignoring small-scale fluctuations. In the present paper only the former definition of coarse-grained drawdowns and drawups will be applied in the analysis, the latter being considered elsewhere [16].

Price coarse-grained drawdowns can be defined as follows. We identify a local maximum in the price and look for a continuation of the downward trend ignoring movements in the reverse direction smaller than ϵ . Here ϵ is referred to as “the threshold”, absolute or relative. Specifically, when a movement in the reverse direction is identified, the drawdown is nevertheless continued *if* the magnitude, absolute or relative, is less than the threshold. A very few drawdowns initiated by this algorithm end up as drawups and are discarded.

In figures 5 to 7 we see the cumulative ϵ -drawdown distributions of the DJIA, SP500 and Nasdaq. The thresholds used were a relative threshold of $\epsilon = 0.01$ for DJIA and Nasdaq and an absolute threshold of $\epsilon = 0.02$ for SP500, which illustrates the problem of an objective determination of what threshold to use. This problem will be addressed in more detail in a future publication elsewhere [16]. We see that the fits with eq. (3) for all three index fully captures the distributions *except* for a few cases which can be referred to as outliers. The dates of these outliers are

- DJIA: 1914, 1987 and 1929
- SP500: 1987, 1962, 1998, 1987, 1974, 1946, 2000
- Nasdaq: 1987, 2000, 1980, 1998, 1998, 1973, 1978, 1987, 1974

Except for the crashes related to the outbreak of WWI in 1914, the Yom Kibbur (Arab-Israeli) war and subsequent OPEC oil embargo in 1973 and the resignation and quite controversial pardoning of president R. Nixon in 1974, quite remarkably all of these outliers have log-periodic power law precursors well-described by eq. (1) and all, except for the Nasdaq crashes of 1978 and 1980, have previously been published [3]. The Nasdaq crashes of 1978 and 1980 was prior to this analysis unknown to us as the “pure”, *i.e.*, no threshold, presented in [1, 2] did not reveal its “outlier nature”.

The results presented here means that the *joint evidence* from the distributions of drawdowns in the DJIA, the SP500 and the Nasdaq identifies all crashes with log-periodic power law precursors found on the US stock market except the crash of 1937². Using $\epsilon = 0.02$ for the Nasdaq reduces the number of obvious outliers to five, two related to the 1987 and 2000 crashes each and one to the 1998 crash. Using $\epsilon = 0.02$ for the DJIA changes nothing whereas this threshold will remove all outliers except the crash of 1987 as outliers and, excluding the historical events of 1914, 1973 and 1974, vice versa.

Naturally, the optimal threshold (according to some specific definition) used in the outlier identification process is related to that particular index volatility. However, the volatility is again nothing but a measure of the two-point correlations present in the index, which we have proven to be a insufficient measure when dealing with extreme market events. Hence, the thresholding procedure proposed and used here is a preliminary step toward more sophisticated amplification tools designed to better capture higher order correlations responsible for the extreme market events [16].

3 Conclusion

The analysis presented here have strengthen the evidence for outliers in the financial markets and that the concept can be used as a objective and quantitative definition of a market crash. Furthermore, we have shown that the existence of outliers in the drawdown distribution is primarily related to the existence of log-periodic power law bubbles prior to the occurrence of these outliers or crashes. In fact, of the 19 large drawdowns identified as outliers only 3 did not have prior log-periodic power law bubble and these 3 outliers could be linked to a specific major historical event. In complement, only 1 (1937) previously identified log-periodic power law bubble was not identified as an outlier.

Further work is needed to clarify the role of different coarse-graining methods as well as to arrive at an objective choice for ϵ . Last, any microscopic market model of log-periodic power law bubbles followed by large crashes should be address the question of why is the mean for $\omega = 2\pi/\ln \lambda$ so close to 2π giving a value for the discrete scaling factor $\lambda \approx e$.

²Increasing the threshold for the DJIA does not improve this miss. Using a temporal coarse-graining as defined previously with a delay of > 2 days attributes a drawdown of $\approx 19\%$ to the crash of 1937 but still places in (the far end of) the bulk of the distribution.

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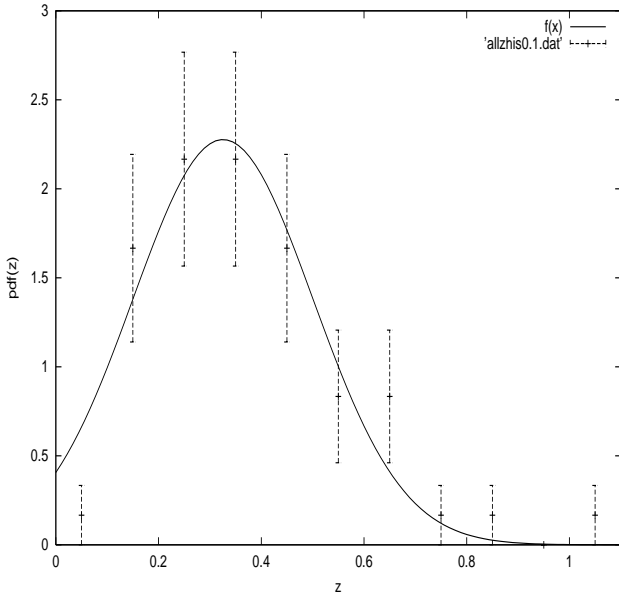


Figure 1: Distribution of fitted exponents z in eq. (1 for over 30 bubbles. The fit is a Gaussian with mean 0.33 and standard deviation 0.18

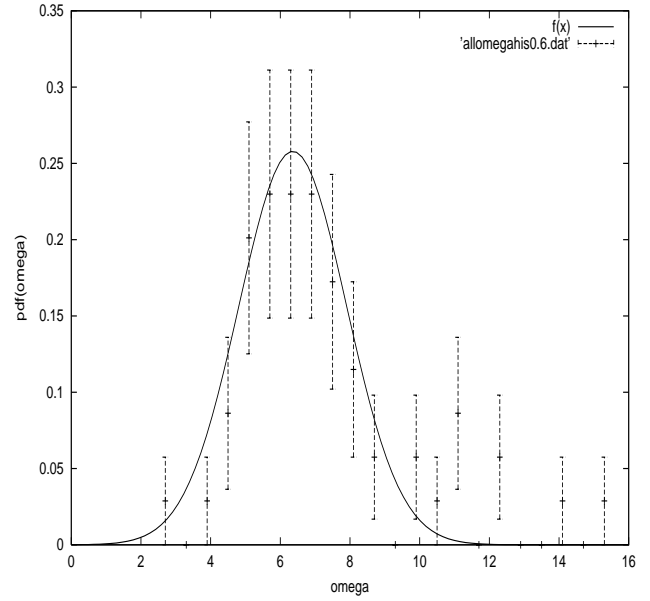


Figure 2: Distribution of fitted log-frequencies ω in eq. (1 for over 30 bubbles. The fit is a Gaussian with mean 6.36 and standard deviation 1.55

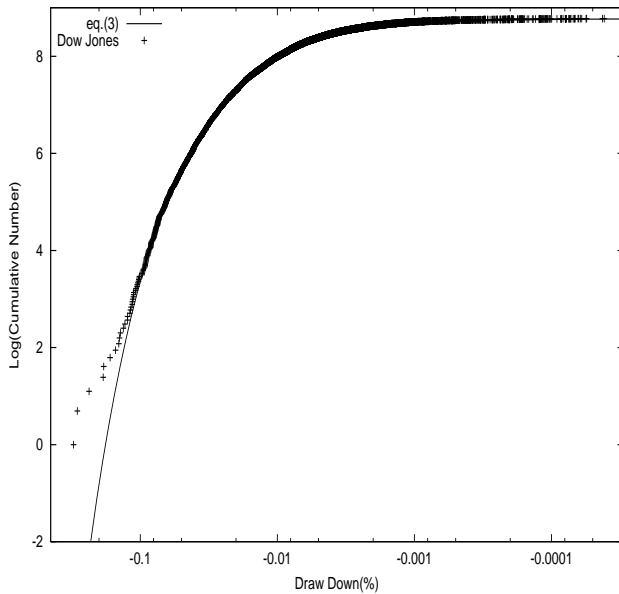


Figure 3: Natural logarithm of the cumulative distribution of drawdowns in the DJIA since 1900 until 2 May 2000. The fit is $\ln(N) = \ln(6469) - 36.3x^{0.83}$, where 6469 is the total number of drawdowns.

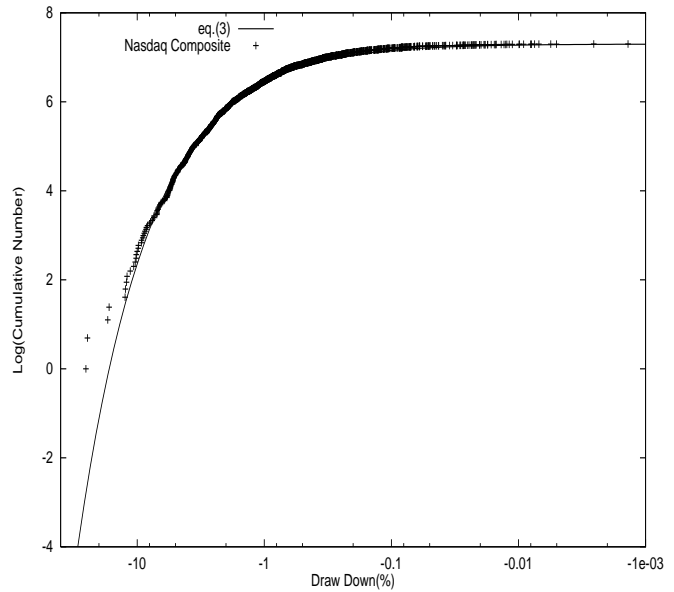


Figure 4: Natural logarithm of the cumulative distribution of drawdowns in the Nasdaq since its establishment in 1971 until 18 April 2000. The fit is $\ln(N) = \ln(1479) - 29.0x^{0.77}$, where 1479 is the total number of draw downs.

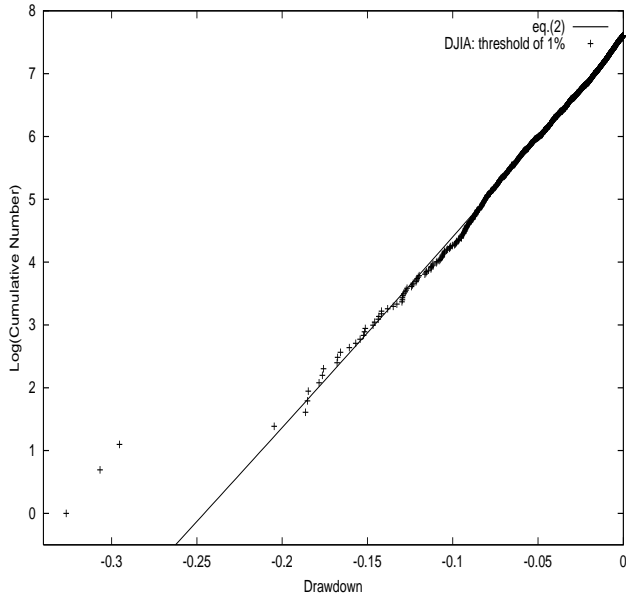


Figure 5: DJIA 2004 events. Natural logarithm of the cumulative distribution of drawdowns coarse-grained with a relative threshold of 0.01. The fit is $\ln(N) \approx 7.60 - 29.4x^{0.96}$.

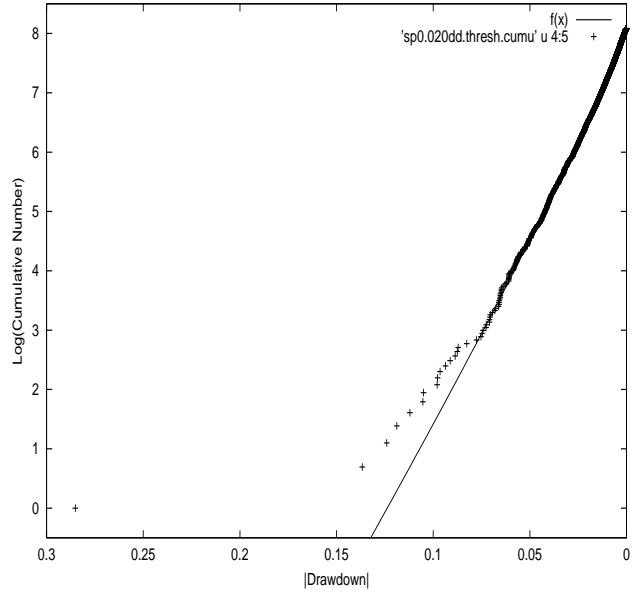


Figure 6: SP500 3239 events. Natural logarithm of the cumulative distribution of drawdowns coarse-grained with an absolute threshold of 0.02. The fit is $\ln(N) \approx 8.08 - 53.3x^{0.90}$.

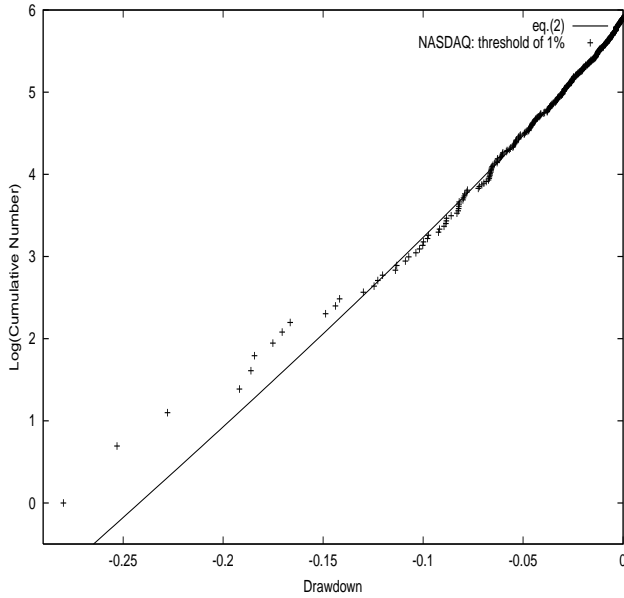


Figure 7: Nasdaq 366 events. Natural logarithm of the cumulative distribution of drawdowns coarse-grained with a relative threshold of 0.01. The fit is $\ln(N) \approx 5.91 - 21.1x^{0.90}$